

# TERM STRUCTURE OF INTEREST RATE. EUROPEAN FINANCIAL INTEGRATION.

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**Abstract:** In this paper we estimate, analyze and compare the term structures of interest rate in six different countries, during the period 1992-2004. We apply Nelson and Siegel model to obtain them with a weekly frequency.

Four European Monetary Union countries: Spain, France, Germany and Italy are included. UK is also included as a European country, but not integrated in the Monetary Union. Finally US completes the analysis.

The goal is to determine the differences in the shape of curves between these countries. Likewise, we can determinate the most usual term structure shapes that appear in every country.

**Keywords:** term structure of interest rate, parsimonious models, level parameter, slope parameter, European interest rate.

**JEL code:** C14, C51, C82, E43, G15

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## 1. Introduction

The capital mobility increasing and the market regulation decreasing are the main features of the current international economic system. This new situation affects directly to the monetary policy decisions. Financial markets provide information about the economic situation and allow anticipating some effects from possible economic decisions. The monetary authorities use variables such as monetary aggregates, interest rate, exchange rate, ... to make monetary policy and pay special attention to the information contained in the yield curve. Among financial indicators, the term structure of interest rates provides a valuable source of information for policymakers.

We define term structure of interest rate as a function of the interest rate related to a specific term. The best resource to obtain the required information is public debt.

Since we can not determinate the interest rate directly from the market for a wide range of maturities, we use different methodologies to obtain it. Professionals choose a model depending on the aim of the study; models based on *splines* have been widely accepted between them. This kind of models fits well the particularities overall the curve. However they usually present explosive tails, that is, long term interest rates are not as asymptotic as it would be desirable.

Most of Central Banks prefer to apply parsimonious functional forms (Anderson et al., 1996), specifically the model proposed by Nelson and Siegel (1987) and the extended version of Svensson (1994). Both are widely used for monetary policy analysis (Bank for International Settlements, 2005). In general, these models smooth the curve but they respect the asymptotic properties.

Table 1 relate the models applied by many Central Banks. The majority of countries apply Nelson and Siegel (NS) or the extended version of Svensson (SV), except countries as Japan,

United Kingdom and United States. However, United Kingdom used the Svensson model from January of 1982 to April of 1998.

The purpose of the paper is to obtain the term structures of interest rates with a theNS model to analyze the term structure of interest rate of the last decade. The objective is to obtain the parameters (level, slope and curvature) of the model and compare the evolution of these curves in the Monetary Union. The study include a period of thirteen years, from 1992 to 2004 and we have analyzed the evolution of these term structures in six different countries: Spain, France, Germany, Italy, United Kingdom and United States. The first four countries are members of the European Monetary Union (EMU). Germany, France and Italy already participated in the creation of the European Monetary System (EMS). Although Spain didn't adhere to the EMS up to 1986, it is interesting to see that, as Italy, from an economic situation very different from France and Germany, it was able to reach the approaches settled down by Maastricht. Moreover, both Italy and Spain are able to be part of the Economic and Monetary Union on first of January in 1999. The evolution of Germany, France, Italy and Spain allows us to analyze the process of convergence of the single currency countries versus United Kingdom and United States, which are a reference to contrast the differences. Furthermore, UK and USA allow us to determinate if the convergence is just in countries with a unique currency or it exists in other financial markets.

We estimate the term structures of interest rates applying the NS model, because of its wide application and international relevance in the monetary policy context.

Section 2 details the elaboration of the data base. In section 3 we define the model. The process used to estimate the term structures is specified in section 4. Section 5 reports the results of comparing the different parameter vectors among countries. Finally, section 6 includes conclusions.

### **Insert Table 1**

## 2. Data base

The most common way to obtain the term structure of interest rates is from national debt, since they present a practically null insolvency risk. According to the Bank for International Settlement (2005), the majority of Central Banks use national debt to obtain their curves.

The required information has mainly been provided by Bloomberg and some Central Banks. Concretely, we have requested the average prices of daily traded national debt and the characteristics of these bonds for every country and all years. It is to say, the ISIN code, the sort of bond, the nominal interest rate, the issued date, the maturity date, the frequency of coupon, as well as the accrual date and the first coupon payment date.

The two fundamental types of public debt are treasury bills and bonds. Generally, these two instruments are a high percentage of the market debt issued by a government. Treasury bills, which do not pay periodic interest, are issued at discounted rate and matures at different term depending on the country. The government bonds pay a fixed interest rate and have a fixed maturity date. All those bonds that present additional features or indexed variable coupons are excluded from the data set. Likewise, benchmark or referenced bonds are used to cover those necessary terms in which any information is provided. Table 2 summarizes the government debt emitted by every Central Bank included.

The elaborated data set contains a total of 1.116.397 references, which means 64.998.548 observations. Figure 1 shows weekly average of bonds over a year given for each country. We can observe the liquidity of the public debt market is different in every country since its issue is related with the financial necessities of a government. The figure 1 presents a summary of the available data for different maturities in the six countries.

**Insert Table 2**

**Insert Figure 1****3. The model**

The NS model define the term structure of instantaneous forward rate. They assume that the instantaneous forward rate is the solution to a second-order differential equation with two equal roots. Hence, NS model establish that the functional form for the instantaneous forward rate at time  $t$  is the following:

$$f_m(\beta) = \beta_0 + \beta_1 \exp\left(\frac{-m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(\frac{-m}{\tau_1}\right), \quad (1)$$

where  $\beta = (\beta_0, \beta_1, \beta_2, \tau_1)$  denotes the vector of the parameters to estimate and  $m$  is the maturity.

Consequently, the spot rate for the time to maturity  $m$ ,  $z_{t,m}$ , is calculated by integrating the forward rate. The discount factor  $\delta_m$  for this period is equal to the exponential term  $\exp(-z_{t,m} \cdot m)$ . So, the discount factor is obtained by applying the previous mathematical relationship:

$$\delta_m(\beta) = \exp\left[-\beta_0 m - (\beta_1 + \beta_2) \tau_1 \left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right) + \beta_2 m \exp\left(-\frac{m}{\tau_1}\right)\right]. \quad (2)$$

The spot rate, the discount factor and the forward rate have convenient properties. The limit:

$$\lim_{m \rightarrow \infty} f_m = \beta_0$$

shows that the forward rate curve converges asymptotically towards the parameter  $\beta_0$ , which can be interpreted as a long-term interest rate. If the maturity approaches zero,

$\lim_{m \rightarrow 0} f_m = \beta_0 + \beta_1$  the forward rate equals the parameter combination  $\beta_0 + \beta_1$ , which can be

interpreted accordingly as a very short rate (instantaneous interest rate). Thus, the forward rates approach a constant for long maturities and settlements. The parameter  $\beta_1$  shows the *spread* between the short and long interest rate. This is an interesting parameter, since variations between the starting and the ending point of the curve generate changes in the slope.

The parameters  $\beta_2$  and  $\tau_1$  do not appear in the very short or long term and do not have a comparable direct interpretation. Since the curvature is shown in intermediate term, they influence the shape of the curve between these limits.

The *forward* function can present a stationary point in  $\frac{m}{\tau_1} = 1 - \frac{\beta_1}{\beta_2}$ , under the condition

$|\beta_1| < |\beta_2|$ . Then, the parameter  $\beta_2$  determines the magnitude and shape of the curvature. If  $\beta_2$  is positive, the curve will have an interior maximum and if  $\beta_2$  is negative, it will exist an interior minimum. When  $\beta_2$  equals zero, monotocidy will occur in the term structure of *forward* rate.

The stationary point will be nearer or further from the value of the parameter  $\beta_0$ , depending on if  $\beta_1$  and  $\beta_2$  are positive or negative. If the shape of the curve is U inverted, it is to say,  $\beta_2 > 0$ , and the condition  $\beta_1 < 0$  is given, then the function starts from a point under  $\beta_0$  and crosses the horizontal asymptote. Otherwise, when  $\beta_1 > 0$ , then the function starts from a point over  $\beta_0$  and it never crosses the asymptote.

Analogous, if the function is U-shaped, that is  $\beta_2 < 0$ , and the condition  $\beta_1 < 0$  is given, then the function starts from a point under  $\beta_0$  and remains under the horizontal asymptote. Otherwise if  $\beta_1 > 0$ , the function falls from a point over  $\beta_0$  and crosses the horizontal asymptote.

The speed to which the instantaneous forward rate comes close to its asymptotic level  $\beta_0$  depends on  $\tau_1$ . An increase in  $\tau_1$  shifts the curvature towards the right, so that the bigger  $\tau_1$  is, the slower the forward interest rate will tend towards  $\beta_0$ . The parameter  $\tau_1$  can just take positive values in order to guarantee the long-term convergence of  $\beta_0$ . Given  $\beta_1$  and  $\beta_2$ ,  $\tau_1$  is determined by  $m$  according to the expression  $\frac{m}{\tau_1} = 1 - \frac{\beta_1}{\beta_2}$  (see Meier 1999 and Schich 1996).

Under the condition  $|\beta_1| \geq |\beta_2|$ , the term structure does not present a stationary point. Then, if the parameter  $\beta_1$  is negative, the term structure is monotonically rising and if  $\beta_1$  is positive, the function will be monotonic falling. In this case, if  $\beta_2 > 0$  and besides  $\beta_1 < 0$ , the second derivate is negative and the function will be concave. On the contrary, if  $\beta_2 < 0$  and  $\beta_1 > 0$ , the second derivate is positive and the function is convex. Otherwise, the concavity or convexity of the function can not be determined since it depends on the value and magnitude of these parameters.

The table 3 summarizes the different shapes of the term structure of interest rate according to the relationships and signs of the different parameters that define the NS model:

Insert Table 3

Thus, it is possible to carry out a descriptive analysis of the term structures of different countries by applying these relationships between parameters.

#### 4. Estimating the term structures

Prior to obtain the term structures, it is necessary to decide the frequency of estimation. It is required that the frequency of estimation reflects all the shifts of the curve. Likewise, we need to guarantee that enough data is provided in every period to estimate the optimum parameters. From the analysis of the daily and weekly distribution data in each one of the countries, we consider that the optimum frequency to estimate the term structures is weekly. With this

frequency, it is guaranteed to represent the evolution of the term structure in every country for the complete analyzed period. So, the total number of curves to estimate rises to 4.038, corresponding to each week between 1992 and 2004,, for each of the six countries.

Before proceeding to the estimation of the parameters, we need to carry out a process of depuration in the data set. The yield-to-maturity curves are a useful instrument before obtaining the parameters of the term structure of interest rates. The yield-to-maturity  $r_{t,i}$  of a bond  $i$  in a moment  $t$  can be calculated with a iterative process from the equation of the price of a bond:

$$P_{t,i} = \sum_{m_i=1}^{M_i} C_i \cdot (1 + r_{t,i})^{-m_i} + N_i \cdot (1 + r_{t,i})^{-M_i} . \quad (3)$$

The yield  $r_{t,i}$  for the maturity  $M_i$  represents the average rate of return from holding a bond for  $M_i$  years, assuming that all coupon payments are reinvested during the maturity of the bond at exactly the same interest rate  $r_{t,i}$ . Depuration process is carried out in order to eliminate the short term outliers. This outliers difficult the estimation of the parameters. Likewise, we eliminate all those observations with negative yield-to-maturity or that ones that remain very far from the yield curve.

We estimate the parameter vector  $\beta_t = (\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \tau_{1,t})$  separately for every week. The non-linear model estimate is:

$$P_{t,i} = C_i \cdot \sum_{m_i=1}^{M_i} \left( \exp \left[ -\beta_{0,t} m_i - (\beta_{1,t} + \beta_{2,t}) \tau_{1,t} \left( 1 - \exp \left( -\frac{m_i}{\tau_{1,t}} \right) \right) + \beta_{2,t} m_i \exp \left( -\frac{m_i}{\tau_{1,t}} \right) \right] \right) + \quad (4)$$

$$+ N_i \cdot \left( \exp \left[ -\beta_{0,t} M_i - (\beta_{1,t} + \beta_{2,t}) \tau_{1,t} \left( 1 - \exp \left( -\frac{M_i}{\tau_{1,t}} \right) \right) + \beta_{2,t} M_i \exp \left( -\frac{M_i}{\tau_{1,t}} \right) \right] \right) + \varepsilon_{t,i},$$

where  $P_{t,i}$  corresponds to the price of a bond  $i$  in a moment  $t$ ,  $C_i$  is the coupon payment,  $N_i$  represents the redemption payment,  $m_i$  are the moments where the different coupon payments



and redemption payment take place and  $\varepsilon_{t,i} \forall t,i$  are random errors, identical and independent distributed (iid), that we assume normal distribution with average 0 and variance  $\sigma_\varepsilon^2$ .

We minimize the sum squared error (SSE) between the observed price ( $P_{i,t}$ ) and the fitted price by the model ( $\hat{P}_{i,t}$ ) to estimate the parameter vector  $\beta_t = (\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \tau_{1,t})$ . The difference between the two prices is weighted by an inversed factor proportional to the maturity. The optimization criterion of the models with parsimonious forms can be defined according to the price or else, according to the yield to maturity. When the minimization of the error in price is applied, a factor inversely proportional to the duration is often included. The inclusion of this factor is due to the difficulty in fitting the short term of the term structure. In general, the fitting in the long term is quite good, since the function is asymptotic by definition. However, the estimated term structure does not fit sometimes very well at the short and mid term, where the slope and curvature are given. As alternative to improve the fitting, it is proposed to use an error on prices weighted by a factor inversely proportional to the duration (Ricart and Sicsic, 1995; Bolder and Stréliski, 1999).

The sum squares weighted error (SSWE) that we apply to obtain the estimated parameter vector corresponds to:

$$SECP_t(\hat{\beta}_t) = \sum_{i=1}^{n_t} \alpha_{t,i} \left( e_{t,i}(\hat{\beta}_t) \right)^2 = \sum_{i=1}^{n_t} \left( \frac{1/d_{t,i}}{\sum_{s=1}^{n_t} 1/d_{t,s}} \right) \left( P_{t,i} - \hat{P}_{t,i} \right)^2, \quad (5)$$

where  $d_{i,t}$  is the duration of a bond  $i$  in a week  $t$  and  $n_t$  is the number of bonds in a week  $t$ . In this case, the duration definition corresponds with the Macaulay modified duration.

A Newton iterative algorithm with SAS 9.1 is used to minimize the objective function. In order to start the iterative algorithm, initials values for the parameter vector  $\beta_t$  have to be

provided. This initial values influences significantly in the final estimated parameter vector. If the initial parameter vector is very different of the optimum one, the algorithm does not converge. In general, the parameter vector of the previous week is suitable to obtain the optimum parameter vector of the following week. However, in periods of instability, the parameters vary between weeks. In those cases it is necessary to find other alternative values to initialize the iterative process.

Since there is a relation between the term structure of interest rates and the yield curve, we use this yield curve as approach to the starting vector of parameters for the first week in 1992. As well as, it is also useful in all those cases where the previous week parameter vector is not suitable.

The interpretation of  $\beta_0$  and  $\beta_1$  makes easy to specify starting values for these parameters.  $\beta_0$  is the long term interest rate implied by the model. Therefore, the smoothed yield with the longest maturity is used as the starting value for  $\beta_0$ . The difference between the smoothed yields with the longest and shortest maturities is used as a starting value of the spread parameter  $\beta_1$ . For  $\beta_2$  or  $\tau_1$  there are no specific economic interpretation. Notwithstanding, when a maximum or minimum exists, and if there is enough data, we can approach the parameter  $\beta_2$  with coordinates  $(r, m)$  as:

$$r = \beta_0 + \beta_2 \cdot \exp\left(-1 + \frac{\beta_1}{\beta_2}\right).$$

$$\frac{m}{\tau_1} = 1 - \frac{\beta_1}{\beta_2}$$

Otherwise, when the yield curve does not present a stationary point or the starting values are not convenient, the estimation is carried out specifying several reasonable starting values. The parameter estimates that provide the best fit are used.

Besides, constraints are imposed on parameters and on spot rate curve in order to exclude implausible and unrealistic estimation results. Thus we refuse negative values in  $\beta_{t,0}$  and  $\tau_{t,1}$ . Likewise, we must guarantee a short-term interest rate, equivalent to the sum of parameters  $\beta_0 + \beta_1$ , is always positive.

Over most of thirteen-year sample period and for all countries considered, the model has produced reliable and reasonable estimation results. Once the 679 parameter vectors for each country have been estimated, we can obtain the forward function, the discount factor or the spot function for any time to maturity  $m$ . From all possible parameter constellations, the term structure shapes are found for every country and week between 1992 and 2004. The analysis of these parameters and the different shapes are one of the main results of this paper.

In the appendix 1, the descriptive analysis of the weekly estimated parameters for every year is presented and is available to consult any weekly term structure of interest rate from January 1992 to December 2004 in <http://guillen.eco.ub.es/~eruizd>.

## 5. Results

The process of estimation, exposed in the previous section, has generated the weekly spot and forward curves for the six countries, from 1992 to the 2004 both included.

According to the values and the relationship between the parameters detailed in table 3, the term structure defined by the NS model can take the different shapes. Thus we are able to apply these shapes to the obtained parameter vectors in each country. The results are detailed in table 4.

It is highlighted that the most frequent forms in the EMU countries are the increasing and the through bellow  $\beta_0$  shapes. These both two shapes are given around of 70% of the total of the 673 analyzed weeks in every country. In the United Kingdom besides these two shapes is also observed the hump crossing  $\beta_0$  shape. This last shape occurs practically only in the United

Kingdom. In United States, beside the increasing and the through bellow  $\beta_0$  shapes, we can find curves with hump that crosses  $\beta_0$ . The detailed estimated curves for each week and country are available in <http://guillen.eco.ub.es/~eruizd>.

#### Insert Table 4

Since we are interested in comparing the time serial of the term structure of interest rate, the first step is to observe the evolution of each parameter in the NS model. In figures 2 to 5 the parameters  $\beta_0, \beta_1, \beta_2$  and  $\tau_1$  are represented. From these figures we realize that the values of the parameters are very similar between EMU countries from 1999, mainly for the  $\beta_0$  and  $\beta_1$  parameters.

Consequently, we would like to contrasting if the term structures of interest rates are really the same or the EMU countries maintain some differences between shape curves.

#### Insert Figure 2, 3, 4 and 5

In order to contrast if term structures of interest rates are equal between these four countries, we apply univariant and multivariant inference. The univariant inference is used for each of the four parameters ( $\beta_0, \beta_1, \beta_2, \tau_1$ ) defined by NS model for the EMU countries. The null hypothesis in the contrast testis defined as equal parameters between countries. It is to say:

$$\begin{aligned} H_0 : \beta_0^{Sp} &= \beta_0^{It} = \beta_0^{Fr} = \beta_0^G \\ H_0 : \beta_1^{Sp} &= \beta_1^{It} = \beta_1^{Fr} = \beta_1^G \\ H_0 : \beta_2^{Sp} &= \beta_2^{It} = \beta_2^{Fr} = \beta_2^G , \\ H_0 : \tau_1^{Sp} &= \tau_1^{It} = \tau_1^{Fr} = \tau_1^G \end{aligned}$$

where ‘Sp’ is Spain, ‘It’ Italy, ‘Fr’ France and ‘G’ Germany.

This inference is made for each year from 1999 to 2004. We calculate the F-Fisher statistic in the variance analysis for each year in order to contrast the null hypothesis. It allows us to compare the differences of the parameters between countries with the differences of the parameters inside countries for each year.

In general, let  $Y_t^i$  be the variable associated to a country  $i$  in time  $t$  (where in our case  $t$  indicate week in a year),  $\bar{Y}^i$  is the average in the country  $i$  and  $\bar{Y}$  is the average for all times and all countries. The sum of squares between countries is:

$$E = \sum_{t=1}^T \sum_{i=1}^N (\bar{Y}^i - \bar{Y})^2 = T \sum_{i=1}^N (\bar{Y}^i - \bar{Y})^2$$

and the sum of squares inside countries is:

$$D = \sum_{t=1}^T \sum_{i=1}^N (Y_t^i - \bar{Y}^i)^2,$$

where  $N$  is the number of countries and  $T$  is the number of times.

The F-Fisher statistic for the equality averages contrast is:

$$F = \frac{T \sum_{i=1}^N (\bar{Y}^i - \bar{Y})^2}{\sum_{t=1}^T \sum_{i=1}^N (Y_t^i - \bar{Y}^i)^2} \frac{T - N}{N - 1},$$

and it is distributed as a F-Fisher with  $(N-1)$  degrees of freedom in the numerator and  $(T-N)$  degrees of freedom in the denominator.

In order to contrast the null hypothesis for each defined parameter, we consider  $Y_t^i$  as an estimated parameter for a country  $i$  in a week  $t$ .

We calculate the F-statistic for each year in the analyzed period. The Fisher contrast is based in the independence hypothesis of the . However, we can not suppose that the estimated parameters in two consecutive weeks are independence. Then we select the first week of each four. This allows us to assume there is independence between observations. We have

calculated the results with the second, the third and the fourth week and they were very similar.

The results obtained for each year between 1999 and 2004 are shown in table 5. We can see that for the years 1999 and 2003 we can not reject that the term structures of interest rates for Germany, France, Italy and Spain are the same. Thus, the term structures interest rates are:

### **Insert Table 5**

statistically

statistically equal for the EMU countries in these two years. Moreover, for years 2000 and 2001 the difference is in  $\tau_1$ . Respect 2002, there exist significant differences among countries in  $\beta_0$  and  $\beta_1$ , although  $\beta_2$  and  $\tau_1$  are statistically equal. Finally, the opposite situation is given in 2004. In general, the  $\beta_0$  and  $\beta_1$  parameter values are not statistically different, so the term structures in the EMU countries present the same level and slope from 1999, except for 2002. Thus, we accept the short and long term interest rates are equals but we reject that the medium term interest rates are the same in these four countries, since the curvature is manifested in the medium term.

To complete the univariate inference, we have calculate the multivariate inference. Now, the null hypothesis is:

$$H_0 : \beta^{Sp} = \beta^{It} = \beta^{Fr} = \beta^G,$$

where  $\beta^i = (\beta_0^i, \beta_1^i, \beta_2^i, \tau_1^i)$  is the parameter vector for a country  $i$ . We calculate the Wilks' Astatistic for the multivariate contrast in each year of period. This statistic is calculated as the following:

$$\Lambda = \frac{|E|}{|T|}$$

where  $|\cdot|$  denote determinant. In this case,  $E$  is a matrix of sum of square between countries and  $T=E+D$ , being  $D$  the matrix of sum of square inside countries, that is:

$$E = \begin{pmatrix} T \sum_{i=1}^N (\bar{\beta}_0^i - \bar{\beta}_0)^2 & T \sum_{i=1}^N (\bar{\beta}_0^i - \bar{\beta}_0)(\bar{\beta}_1^i - \bar{\beta}_1) & T \sum_{i=1}^N (\bar{\beta}_0^i - \bar{\beta}_0)(\bar{\beta}_2^i - \bar{\beta}_2) & T \sum_{i=1}^N (\bar{\beta}_0^i - \bar{\beta}_0)(\bar{\tau}_1^i - \bar{\tau}_1) \\ T \sum_{i=1}^N (\bar{\beta}_1^i - \bar{\beta}_1)(\bar{\beta}_0^i - \bar{\beta}_0) & T \sum_{i=1}^N (\bar{\beta}_1^i - \bar{\beta}_1)^2 & T \sum_{i=1}^N (\bar{\beta}_1^i - \bar{\beta}_1)(\bar{\beta}_2^i - \bar{\beta}_2) & T \sum_{i=1}^N (\bar{\beta}_1^i - \bar{\beta}_1)(\bar{\tau}_1^i - \bar{\tau}_1) \\ T \sum_{i=1}^N (\bar{\beta}_2^i - \bar{\beta}_2)(\bar{\beta}_0^i - \bar{\beta}_0) & T \sum_{i=1}^N (\bar{\beta}_2^i - \bar{\beta}_2)(\bar{\beta}_1^i - \bar{\beta}_1) & T \sum_{i=1}^N (\bar{\beta}_2^i - \bar{\beta}_2)^2 & T \sum_{i=1}^N (\bar{\beta}_2^i - \bar{\beta}_2)(\bar{\tau}_1^i - \bar{\tau}_1) \\ T \sum_{i=1}^N (\bar{\tau}_1^i - \bar{\tau}_1)(\bar{\beta}_0^i - \bar{\beta}_0) & T \sum_{i=1}^N (\bar{\tau}_1^i - \bar{\tau}_1)(\bar{\beta}_1^i - \bar{\beta}_1) & T \sum_{i=1}^N (\bar{\tau}_1^i - \bar{\tau}_1)(\bar{\beta}_2^i - \bar{\beta}_2) & T \sum_{i=1}^N (\bar{\tau}_1^i - \bar{\tau}_1)^2 \end{pmatrix}$$

and

$$D = \begin{pmatrix} \sum_{t=1}^T \sum_{i=1}^N (\beta_{0t}^i - \bar{\beta}_0^i)^2 & \sum_{t=1}^T \sum_{i=1}^N (\beta_{0t}^i - \bar{\beta}_0^i)(\beta_{1t}^i - \bar{\beta}_1^i) & \sum_{t=1}^T \sum_{i=1}^N (\beta_{0t}^i - \bar{\beta}_0^i)(\beta_{2t}^i - \bar{\beta}_2^i) & \sum_{t=1}^T \sum_{i=1}^N (\beta_{0t}^i - \bar{\beta}_0^i)(\tau_{1t}^i - \bar{\tau}_1^i) \\ \sum_{t=1}^T \sum_{i=1}^N (\beta_{1t}^i - \bar{\beta}_1^i)(\beta_{0t}^i - \bar{\beta}_0^i) & \sum_{t=1}^T \sum_{i=1}^N (\beta_{1t}^i - \bar{\beta}_1^i)^2 & \sum_{t=1}^T \sum_{i=1}^N (\beta_{1t}^i - \bar{\beta}_1^i)(\beta_{2t}^i - \bar{\beta}_2^i) & \sum_{t=1}^T \sum_{i=1}^N (\beta_{1t}^i - \bar{\beta}_1^i)(\tau_{1t}^i - \bar{\tau}_1^i) \\ \sum_{t=1}^T \sum_{i=1}^N (\beta_{2t}^i - \bar{\beta}_2^i)(\beta_{0t}^i - \bar{\beta}_0^i) & \sum_{t=1}^T \sum_{i=1}^N (\beta_{2t}^i - \bar{\beta}_2^i)(\beta_{1t}^i - \bar{\beta}_1^i) & \sum_{t=1}^T \sum_{i=1}^N (\beta_{2t}^i - \bar{\beta}_2^i)^2 & \sum_{t=1}^T \sum_{i=1}^N (\beta_{2t}^i - \bar{\beta}_2^i)(\tau_{1t}^i - \bar{\tau}_1^i) \\ \sum_{t=1}^T \sum_{i=1}^N (\tau_{1t}^i - \bar{\tau}_1^i)(\beta_{0t}^i - \bar{\beta}_0^i) & \sum_{t=1}^T \sum_{i=1}^N (\tau_{1t}^i - \bar{\tau}_1^i)(\beta_{1t}^i - \bar{\beta}_1^i) & \sum_{t=1}^T \sum_{i=1}^N (\tau_{1t}^i - \bar{\tau}_1^i)(\beta_{2t}^i - \bar{\beta}_2^i) & \sum_{t=1}^T \sum_{i=1}^N (\tau_{1t}^i - \bar{\tau}_1^i)^2 \end{pmatrix}$$

There exists a relation between the Wilks'  $\Lambda$  statistic and the F-Fisher statistic (Rao, 1951).

Table 6 illustrates the results that we have obtained with the multivariant inference.

This results show that we reject the null hypothesis that the parameter vectors are the same in the EMU countries in all years except for 1999. This year reflects the effort made for

the four countries make to reach the approaches settled down by Maastricht and consequently the term structure of interest rates along all this year take the same shape.

### Insert Table 6

## 6. Conclusions

In this paper we estimate, analyze and compare weekly term structure of interest rate from six countries, four EMU countries, Spain, France, Germany and Italy. United Kingdom is

included as a European country not integrated in the EMU and United States as a reference in the international financial markets. The time period are 13 years, from 1992 to 2004, both included.

Previous, the parameters of the model of NS are estimated minimizing the sum squared error in price . A total of 4.038 curves are obtained.

We get two results level. First, we analyse the vector parameter  $\beta_t = (\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \tau_{1,t})$  for each week and country, and with the relationships between parameters summarized in table 3, we establish the different shapes of the term structure of interest rates. The most frequent forms in the EMU countries are the increasing and the through bellow  $\beta_0$  shapes, around of 70% of the total. Only in the United Kingdom is also observed the hump crossing  $\beta_0$  shape. In United States, beside the increasing and the through bellow  $\beta_0$  shapes, we can find curves with hump that crosses  $\beta_0$ .

The second result is related with the difference of the parameters between countries from 1999 to 2004. In this period, the level and the slope parameter show similar values (figures 2 and 3). The univariant contrast reveal that we can accept similary curves for Germany, France, Italy and Spain in the years 1999 and 2003. However, there is no significant difference in level, slope and curvature parameters in 2000 and 2001.  $\beta_0$  and  $\beta_1$  parameter values are different in 2002, although  $\beta_2$  and  $\tau_1$  are statistically equal. Finally, the opposite situation is given in 2004. In general, the  $\beta_0$  and  $\beta_1$  parameter values are coincidents statistically, so the term structures in the EMU countries present the same level and slope from 1999 to 2004, except for 2002. Thus, we accept the short and long term interest rates are equals but we reject that the medium term interest rates are the same in these four countries. The Wilks'  $\Lambda$  statistic for the multivariant contrast in each year of period, shows the parameter vectors are not the same in the EMU countries, except for 1999.



## **Acknowledgements**

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# Appendix 1

## Average and deviation of the parameters

		Germany	France	Spain	Italy	United Kingdom	United States
1992	beta 0	0.07371086 <i>0.00142222</i>	0.08838092 <i>0.00434239</i>	0.10015208 <i>0.013906</i>	0.1196958 <i>0.00568269</i>	0.10065034 <i>0.00462974</i>	0.07976331 <i>0.00319536</i>
	beta 1	0.09449418 <i>0.00367855</i>	0.10289887 <i>0.00442534</i>	0.1321648 <i>0.00926355</i>	0.14023015 <i>0.01847026</i>	0.09316272 <i>0.01461283</i>	0.03306322 <i>0.00384309</i>
	beta 2	0.00484996 <i>0.01867941</i>	-0.011219 <i>0.02461469</i>	0.01495299 <i>0.0195769</i>	0.01847716 <i>0.02393766</i>	-0.0259119 <i>0.02745378</i>	0.05675558 <i>0.01132018</i>
	tau 1	0.97159862 <i>0.22127982</i>	1.078838 <i>0.66197065</i>	4.26874898 <i>2.1561845</i>	2.2162232 <i>7.289596</i>	0.76422489 <i>0.48323933</i>	5.67696487 <i>1.60566231</i>
1993	beta 0	0.06990486 <i>0.00196337</i>	0.0824119 <i>0.00604893</i>	0.10761204 <i>0.00974947</i>	0.11397649 <i>0.01225823</i>	0.09355741 <i>0.01021421</i>	0.07103213 <i>0.00494754</i>
	beta 1	0.07501203 <i>0.00837738</i>	0.08246705 <i>0.01813551</i>	0.12197596 <i>0.01954236</i>	0.10268545 <i>0.01246528</i>	0.05561954 <i>0.00430038</i>	0.02924474 <i>0.00142393</i>
	beta 2	-0.0489967 <i>0.00940503</i>	-0.0622998 <i>0.01018897</i>	-0.0379996 <i>0.0298678</i>	0.00422122 <i>0.02676646</i>	-0.0531599 <i>0.01412124</i>	0.05210225 <i>0.00989326</i>
	tau 1	1.24139298 <i>0.31862643</i>	1.72940174 <i>0.34051903</i>	1.32137731 <i>0.53018412</i>	2.21687055 <i>0.94087192</i>	1.14697764 <i>0.29823645</i>	7.5872027 <i>0.99875159</i>
1994	beta 0	0.07752021 <i>0.00427188</i>	0.08280443 <i>0.00616698</i>	0.10542625 <i>0.00947</i>	0.1109108 <i>0.0106627</i>	0.08786309 <i>0.00576409</i>	0.07584461 <i>0.00651932</i>
	beta 1	0.05162105 <i>0.0060849</i>	0.0564736 <i>0.00400902</i>	0.07795709 <i>0.00660356</i>	0.08752314 <i>0.00424947</i>	0.0500257 <i>0.00366921</i>	0.0435105 <i>0.00823518</i>
	beta 2	-0.0345397 <i>0.01161094</i>	-0.0235886 <i>0.02100524</i>	0.00429538 <i>0.03842263</i>	0.02047093 <i>0.01929772</i>	-0.0405104 <i>0.00783472</i>	0.03377371 <i>0.01713895</i>
	tau 1	1.19589304 <i>0.35118596</i>	1.47190282 <i>0.53026878</i>	1.55454074 <i>0.3297322</i>	3.85419862 <i>3.031062</i>	0.50010591 <i>0.2703182</i>	3.79815113 <i>1.80821227</i>
1995	beta 0	0.07991234 <i>0.00128798</i>	0.08748219 <i>0.00205055</i>	0.11536718 <i>0.00535141</i>	0.12385152 <i>0.00611115</i>	0.09103426 <i>0.0020645</i>	0.0746745 <i>0.00386964</i>
	beta 1	0.04408074 <i>0.00278009</i>	0.06366144 <i>0.00804701</i>	0.09426087 <i>0.00721348</i>	0.10146719 <i>0.0070543</i>	0.06170307 <i>0.00305199</i>	0.05692955 <i>0.00259231</i>
	beta 2	-0.0388052 <i>0.01739502</i>	-0.0326837 <i>0.01935809</i>	0.00074388 <i>0.03278327</i>	0.00183558 <i>0.01985253</i>	-0.0234214 <i>0.01496932</i>	-0.0130138 <i>0.00926378</i>
	tau 1	1.26809548 <i>0.22954746</i>	1.43737477 <i>0.28718239</i>	1.11548011 <i>0.56962115</i>	2.08766085 <i>2.05580913</i>	0.88125312 <i>0.46439149</i>	2.43068784 <i>1.45379101</i>
1996	beta 0	0.07831796 <i>0.00223476</i>	0.08426352 <i>0.00192597</i>	0.09819263 <i>0.00766136</i>	0.1049909 <i>0.00846549</i>	0.08787254 <i>0.00371227</i>	0.07143687 <i>0.00227932</i>
	beta 1	0.03276608 <i>0.00217398</i>	0.03460449 <i>0.00401943</i>	0.07780811 <i>0.00820318</i>	0.07948699 <i>0.0109219</i>	0.05868467 <i>0.00400658</i>	0.05152724 <i>0.00153928</i>
	beta 2	-0.0553614 <i>0.00828567</i>	-0.0019622 <i>0.01405251</i>	-0.043282 <i>0.00761734</i>	0.00204155 <i>0.01105892</i>	-0.0090896 <i>0.02587461</i>	-0.0075477 <i>0.01216173</i>
	tau 1	1.36620582 <i>0.1087861</i>	3.48500571 <i>0.77837546</i>	1.37476982 <i>0.38419047</i>	4.9334002 <i>1.57698254</i>	2.10595946 <i>1.12506409</i>	2.02687205 <i>1.25073874</i>
1997	beta 0	0.07156695 <i>0.003934</i>	0.07507311 <i>0.00626527</i>	0.07720743 <i>0.00678252</i>	0.08108679 <i>0.00427686</i>	0.07335496 <i>0.00699072</i>	0.06900675 <i>0.00241436</i>
	beta 1	0.03332369 <i>0.0019011</i>	0.03282712 <i>0.00279879</i>	0.05607594 <i>0.00390315</i>	0.06046589 <i>0.00484622</i>	0.06562827 <i>0.00426476</i>	0.0533323 <i>0.00200056</i>

1998	beta 2	-0.0335415 0.02552283	-0.0168361 0.01957171	-0.0468774 0.00939762	-0.0203422 0.01116403	0.00704948 0.00912261	-0.0031486 0.007144
	tau 1	2.25547914 0.94777718	3.18718309 1.34602191	1.65361676 0.16590728	3.26258112 1.44294432	1.72115131 1.41506067	2.48175508 2.03779584
	beta 0	0.06006525 0.00131339	0.06105371 0.00135634	0.06184466 0.00222756	0.06194938 0.00266643	0.05307159 0.0052102	0.06016004 0.00203186
	beta 1	0.03458379 0.00180107	0.03629852 0.00162233	0.03936029 0.00490846	0.0464027 0.00627057	0.07101809 0.00429432	0.05212811 0.00234107
	beta 2	-0.0073851 0.01805302	-0.0150109 0.01429201	-0.0251864 0.01073025	-0.0314849 0.0085891	0.01537151 0.00996749	-0.0147364 0.01263188
1999	tau 1	5.24126749 1.48760764	4.21500524 1.13189881	2.72415952 0.72148315	2.71864077 0.96267928	0.80208333 0.31803517	2.36029996 1.16066974
	beta 0	0.06247436 0.00165276	0.06306985 0.00294321	0.06432582 0.00316815	0.062759 0.00146612	0.03996706 0.00560065	0.06367364 0.00141982
	beta 1	0.02929593 0.00320159	0.0292854 0.00340385	0.02732404 0.00348167	0.02875766 0.0028537	0.05116721 0.0038753	0.04920792 0.00191149
	beta 2	-0.0197248 0.01289792	-0.0217319 0.01358005	-0.0088411 0.01511659	-0.0171246 0.01891058	0.03443916 0.03238355	0.00089957 0.01399857
	tau 1	3.18233874 1.03849443	3.05659462 0.64362141	3.97430812 1.76766588	2.71406424 1.11177352	2.4909761 0.83742148	2.6326028 0.94078814
2000	beta 0	0.06082846 0.00292171	0.0617397 0.00232076	0.0619012 0.00300414	0.06242637 0.00183478	0.03524671 0.00210112	0.06151974 0.00260587
	beta 1	0.04688296 0.00614783	0.04595463 0.00620968	0.04483653 0.00640414	0.04629705 0.00660243	0.05941436 0.00200675	0.06117269 0.00369028
	beta 2	-0.0077033 0.01093717	-0.0078245 0.01035339	-0.0011072 0.00639814	-0.0012551 0.01301461	0.03717097 0.01623702	0.00585217 0.01702843
	tau 1	4.15680164 1.52431432	3.61378911 0.94228842	3.56117422 1.16059161	2.66802368 0.9167527	3.50914318 0.78740665	1.1892144 0.72749886
	beta 0	0.06336994 0.00315572	0.06363558 0.00268308	0.06309503 0.00275842	0.06488708 0.00236196	0.04378319 0.00478593	0.06208784 0.00197635
2001	beta 1	0.04029884 0.00627688	0.04112215 0.00596474	0.04034163 0.00496195	0.04019583 0.00587885	0.04813574 0.00545594	0.03560195 0.01153195
	beta 2	-0.0246722 0.007786	-0.0286502 0.00852395	-0.0266789 0.00922029	-0.0263602 0.00781231	0.00619244 0.01616879	-0.0357196 0.01241431
	tau 1	3.47695255 0.78866985	2.7814723 0.38012603	2.43714693 0.75167605	2.16120384 0.57586854	3.59641275 2.55723938	1.48170415 0.51928568
	beta 0	0.05750121 0.00119263	0.05735725 0.00146467	0.05875757 0.00197195	0.05979687 0.00140155	0.04701379 0.0016198	0.06322957 0.00194081
	beta 1	0.03287399 0.00301039	0.03186089 0.00161234	0.03226529 0.00187435	0.03106096 0.00138083	0.03896145 0.0016531	0.01678697 0.00209779
2002	beta 2	-0.018226 0.01286741	-0.017765 0.01559262	-0.0210933 0.01667668	-0.023552 0.00918516	0.01096389 0.02376458	-0.0536378 0.01028689
	tau 1	1.94556499 0.58599232	1.79258842 0.4587025	1.74461792 0.514673	1.34933709 0.4448862	1.89134793 1.17430261	1.24767323 0.49247668
	beta 0	0.05716799 0.00140579	0.05708945 0.0013913	0.05700261 0.00137872	0.05766184 0.00172194	0.04872277 0.00148184	0.06344306 0.00309212
	beta 1	0.02164779 0.00364276	0.02158897 0.00376235	0.02169311 0.00407178	0.02156364 0.00412994	0.03574959 0.00234349	0.01036652 0.00221523
	beta 2	-0.0284172 0.01782092	-0.02716 0.02058286	-0.030786 0.02090119	-0.0304344 0.01864066	-0.01868 0.00949116	-0.0543566 0.01024459
2003	tau 1	2.4451144	2.51410344	2.32393256	2.00612935	0.88000235	1.87998297

2004		<i>0.59758556</i>	<i>0.77269125</i>	<i>0.71621549</i>	<i>0.80488499</i>	<i>0.34546377</i>	<i>0.22780485</i>
	beta 0	0.05582698 <i>0.00231794</i>	0.05570261 <i>0.00205938</i>	0.05674936 <i>0.00229606</i>	0.05739823 <i>0.00285811</i>	0.0438102 <i>0.0038686</i>	0.06294541 <i>0.00170569</i>
	beta 1	0.01951538 <i>0.0011985</i>	0.01858605 <i>0.0008588</i>	0.01756475 <i>0.00124147</i>	0.01865637 <i>0.00115645</i>	0.04267371 <i>0.00369618</i>	0.01413227 <i>0.00520992</i>
	beta 2	-0.0224215 <i>0.00757202</i>	-0.0165416 <i>0.0131045</i>	-0.0006421 <i>0.00509345</i>	-0.0217033 <i>0.02009585</i>	0.01378239 <i>0.01345427</i>	-0.0286783 <i>0.01479081</i>
	tau 1	2.46501117 <i>0.31730944</i>	2.8435505 <i>1.01506694</i>	3.95673001 <i>0.45545746</i>	2.58235042 <i>1.59551113</i>	3.91497532 <i>2.60803719</i>	2.50191505 <i>0.71668594</i>

## References

Anderson, N., Breedon, F., Deacon, M., Derry, A. and Murphy, G., 1996. Estimating and Interpreting the Yield Curve.: John Wiley & Sons Ltd., New Jersey.

Bolder, D. y Stréliski, D., 1999. Yield Curve Modelling at the Bank of Canada. Bank of Canada, Technical Report 84, Febrero.

Greene, W.H., 2003. Econometric analysis. 5th ed. Prentice Hall, cop.

Meier, I., 1999. Estimating the Term Structure of Interest Rates: the Swiss Case. Swiss National Bank. Study Center Garzensee, Working paper, 99.06.

Nelson, C.R., Siegel, A.F., 1987. Parsimonious Modeling of Yield Curves. Journal of Business 60 (3), 473-489.

Rao, C.R., 1951. An asymptotic expansion of the distribution of Wilk's criterium. Bulletin Inst. Inter. Statistics, XXXIII, 2, 177-180.

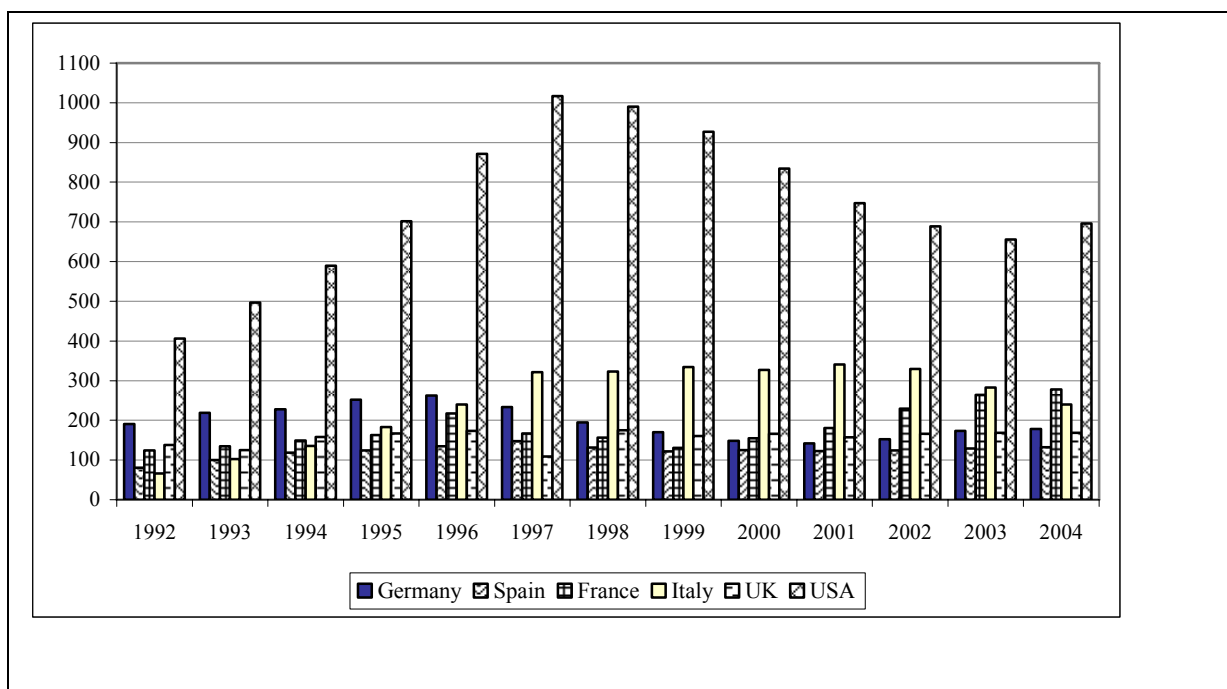
Ricart, R. y Sicsic, P., 1995. Estimating the Term Structure of Interest Rates from French Data. Bulletin Digest, Banque de France, Working Paper 22, 473-489.

Schich, S.T., 1996. Alternative Specifications of the German Terms Structure and its Information Content Regarding Inflation. Economic Research Group of the Deutsche Bundesbank. Discussion paper, 8 October.

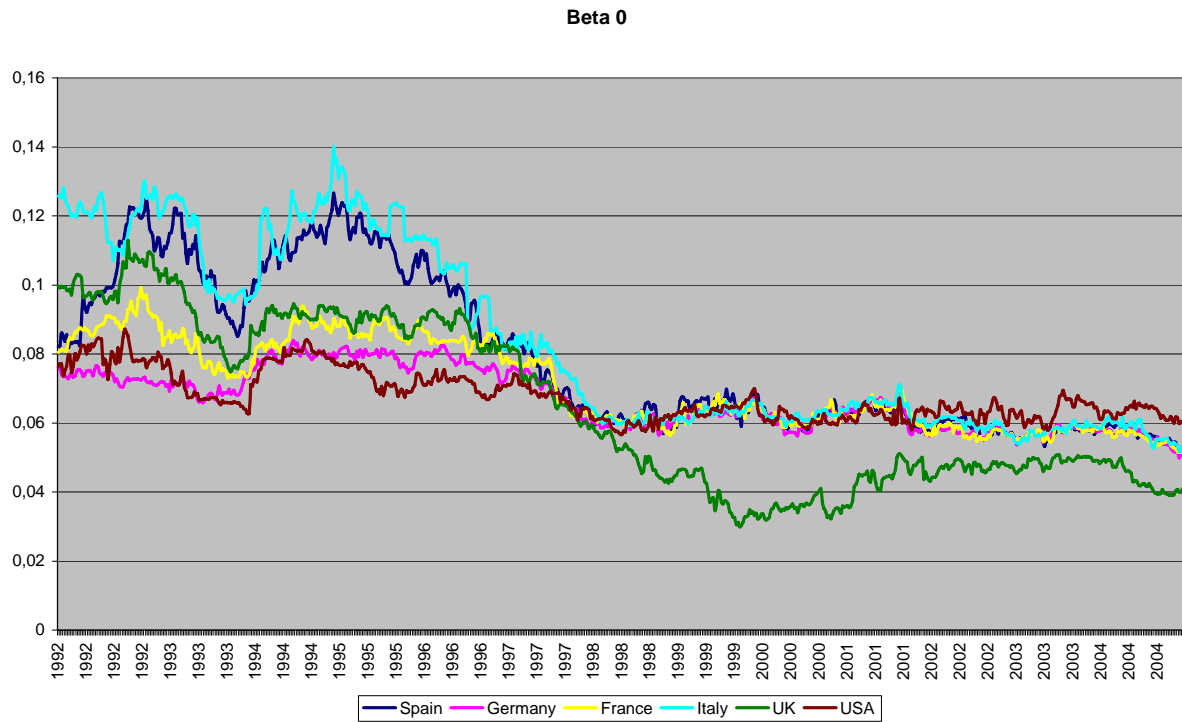
Svensson, L.E.O., 1994. Estimation of Forward Interest Rates. Quarterly Review, Sveriges Riskbank, 3, 33-43.

## Figures

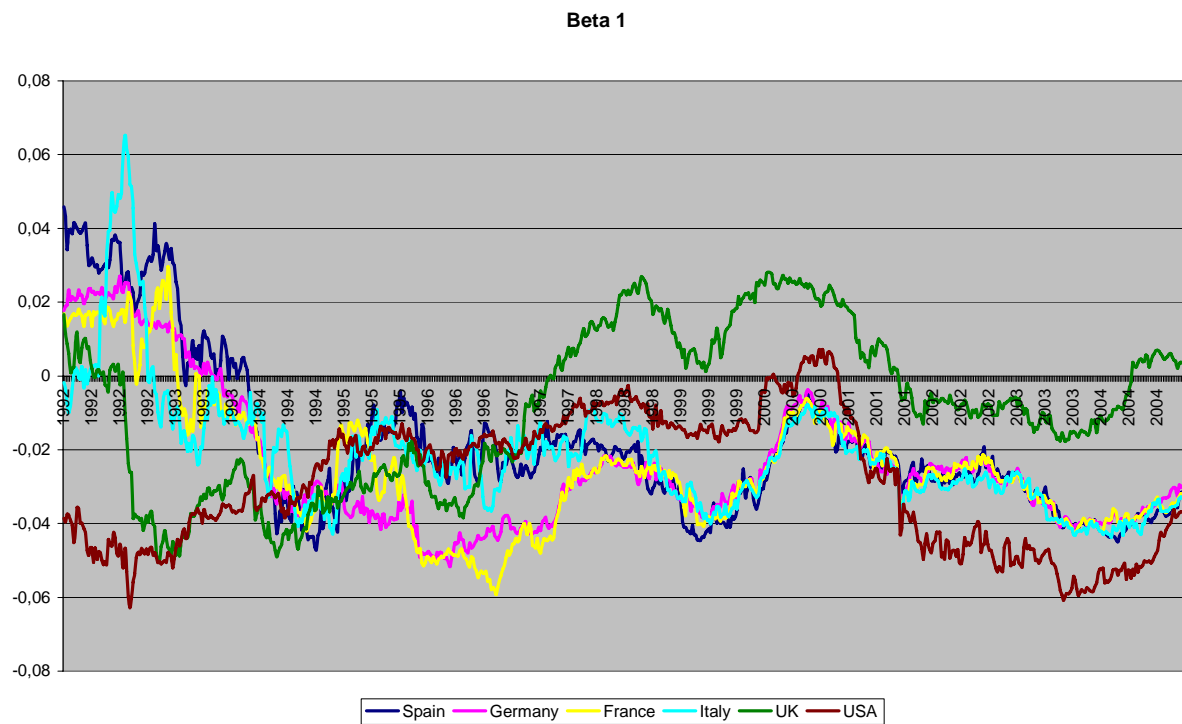
**Figure 1.** Weekly averages for bonds over a year.



**Figure 2.** Weekly level parameter ( $\beta_0$ ) values from January 1992 to December 2004.

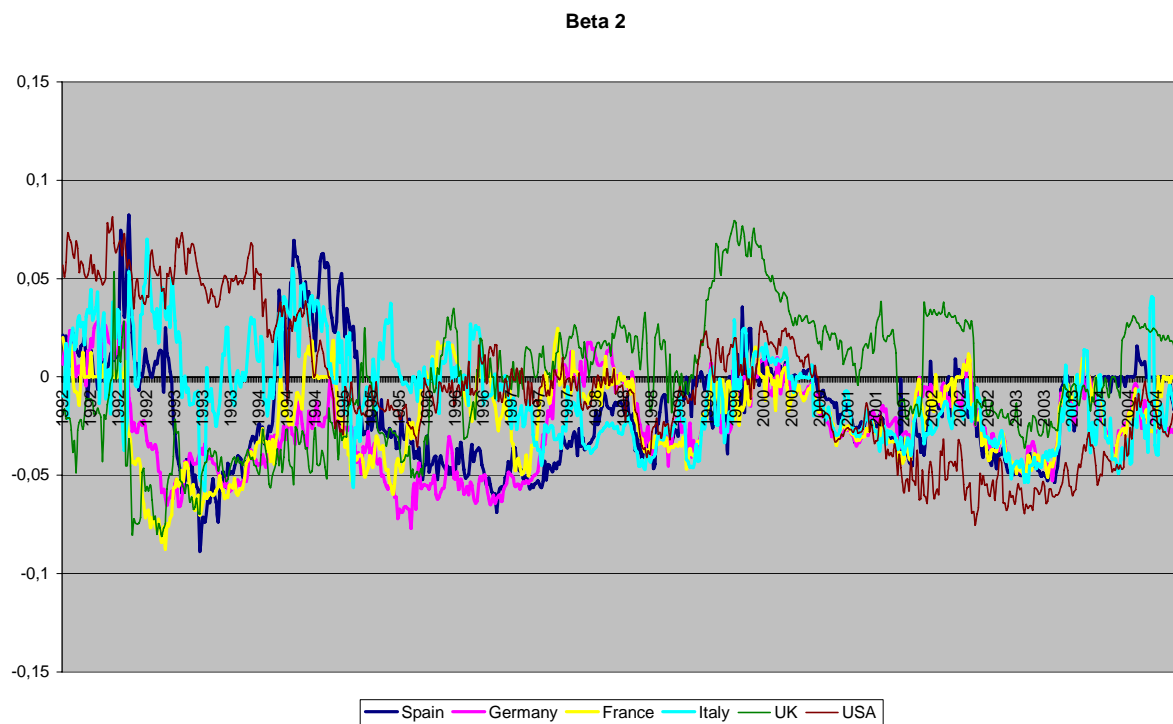


**Figure 3.** Weekly slope parameter ( $\beta_1$ ) values from January 1992 to December 2004.

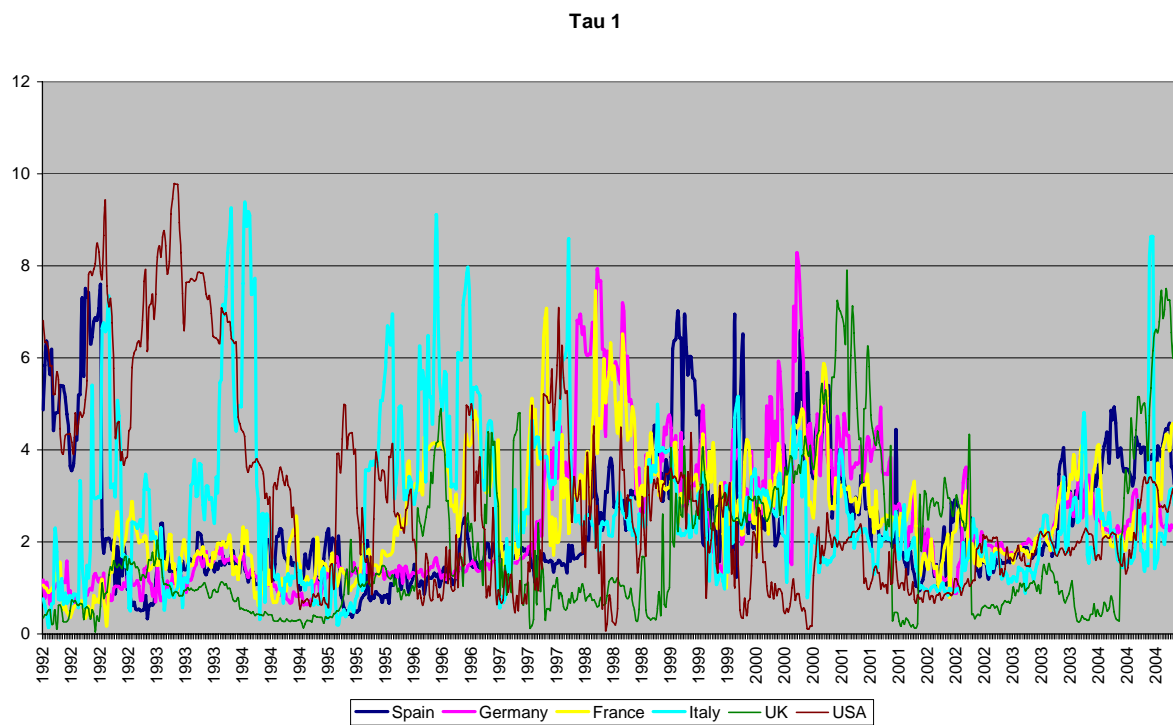




**Figure 4.** Weekly parameter  $\beta_2$  values from January 1992 to December 2004.



**Figure 5.** Weekly  $\tau_1$  parameter values from January 1992 to December 2004.



## **Tables**

**Table 1.** Methodologies applied by Central Banks.

Source: Bank for International Settlements 2005. Zero-Coupon Yield Curves: Technical Documentation. Monetary and Economic Department.

Central Bank	Estimation method <sup>A</sup>	Estimates available since	Frequency	Minimised error	Adjustments of tax distortions	Relevant maturity spectrum
Belgium	SV NS	1 Sept 1997	daily	Weighted prices	No	Couple of days to 16 years
Canada <sup>B</sup>	SV	23 Jun 1998	daily	Weighted prices	Effectively by excluding bonds	1 to 30 years
Finland	NS	3 Nov 1998	weekly; daily from 4 Jan 1999	Weighted prices	No	1 to 12 years
France	SV NS	3 Jan 1992	weekly	Weighted prices	No	Up to 10 years
Germany	SV	7 Aug 1997 Jan 1973	daily monthly	Yields	No	1 to 10 years
Italy	NS	1 Jan 1996	daily	Weighted prices	No	Up to 30 y. Up to 10 y. (bef. Feb.02)
Japan	SS	29 Jul 1998 up to 19 Apr 2000	weekly	Prices	Effectively by price adjustments for bills	1 to 10 years
Norway	SV	21 Jan 1998	± monthly	Yields	No	Up to 10 years
Spain	SV	Jan 1995	daily	Weighted prices	No	Up to 10 y. Up to 10 y.
	NS	Jan 1991	monthly			
Sweden	SV	9 Dec 1992	at least once a week	Yields	No	Up to 10 years
Switzerland	SV	4 Jan n 1998	daily	Yields	No	1 to 30 years
	SV	Jan 1998	monthly			
United Kingdom <sup>C</sup>	SV	4 Jan 1982 up to 30 Ap.1998	daily monthly	Weighted prices	No	1 week to 30 years
	VRP VRP	4 Jan 1982 15 Jan 1985	daily daily	Weighted prices	No	1 week to 30 years
United States	SS	14 Jun 1961	Daily	Bills: Weighted p. Bonds p.	No	Up to 1 year

<sup>A</sup> NS = Nelson-Siegel, SV = Svensson, SS = suavizados splines, VRP = variable de penalización por rugosidad.

<sup>B</sup> Canada está en proceso de revisión de su actual metodología de estimación.

<sup>C</sup> Reino Unido usó el modelo de Svensson entre Enero del 1982 y Abril del 1998

**Table 2.** Public debt issued by every government.

	<b>Instrument</b>	<b>Maturity date</b>	<b>Issued</b>
Germany <i>(Deutschen Finanzagentur)</i>	Bubills	6 months	Discount
	Bobls	5 years	Annual coupon
	Bunds	10-30 years	Annual coupon
Spain <i>(Departamento del Tesoro)</i>	Letras Tesoro	6, 12 and 18 months	Discount
	Bonos del Estado	3 or 5 years	Annual coupon
	Obligaciones del Estado	15 or 30 years	Annual coupon
France <i>(Agence France Trésor)</i>	BTF	3, 6 and 12 months	Discount
	BTAN	From 2 to 5 years	Annual coupon
	OAT	From 7 to 30 years	Annual coupon
Italy <i>(Dipartimento del Tesoro)</i>	BOT	3, 6 and 12 months	Discount
	CTZ	18 or 24 months	Discount
	BPT	3, 5, 10 and 30 years	Semi-annual coupon
United Kingdom <i>(Debt Management Office)</i>	T-bills	1, 3, 6 and 12 months	Discount
	Conventional gilts	5, 10 and 30 years	Semi-annual coupon
United States of America <i>(Bureau of the Public Debt)</i>	T-bills	4, 13 and 26 weeks	Discount
	T-notes	2, 3, 5 and 10 years	Semi-annual coupon
	T-bonds	between 10 and 30 years	Semi-annual coupon

**Table 3.** Term structures shapes and relation between parameters.

Shape	$\beta_0$	$\beta_1$	$\beta_2$	$\tau_1$	Condition
Increasing, concave	+	−	+	+	$ \beta_1  \geq  \beta_2 $
Increasing	+	−	−	+	$ \beta_1  \geq  \beta_2 $
Decreasing, convex	+	+	−	+	$ \beta_1  \geq  \beta_2 $
Decreasing	+	+	+	+	$ \beta_1  \geq  \beta_2 $
Hump, above $\beta_0$	+	+	+	+	$ \beta_1  <  \beta_2 $
Hump, crosses $\beta_0$	+	−	+	+	$ \beta_1  <  \beta_2 $
Through, below $\beta_0$	+	−	−	+	$ \beta_1  <  \beta_2 $
Through, crosses $\beta_0$	+	+	−	+	$ \beta_1  <  \beta_2 $

**Table 4.** Frequency of the term structure shapes.

	Germany	Spain	France	Italy	United Kingdom	United States
Increasing, concave	8%	13%	13%	15%	5%	15%
Increasing	40%	28%	40%	37%	17%	34%
Decreasing, convex	0%	2%	2%	2%	2%	0%
Decreasing	4%	7%	3%	2%	7%	1%
Hump, above $\beta_0$	1%	1%	0%	3%	26%	1%
Hump, crosses $\beta_0$	0%	4%	0%	10%	8%	21%
Through, below $\beta_0$	38%	39%	37%	32%	30%	32%
Through, crosses $\beta_0$	8%	6%	5%	0%	4%	2%

**Table 5.** The F-Fisher statistics results.

	$\beta_0$	$\beta_1$	$\beta_2$	$\tau_1$
<b>1999</b>	1.30	2.44	2.31	1.70
<b>2000</b>	0.83	0.36	1.77	3.21*
<b>2001</b>	1.35	0.67	0.53	10.94*
<b>2002</b>	8.33*	8.62*	0.24	1.65
<b>2003</b>	0.61	0.09	0.16	2.05
<b>2004</b>	1.64	2.61	7.88*	5.44*
<b>All</b>	<b>2.1</b>	<b>0.82</b>	<b>1.90</b>	<b>5.41*</b>

\*Indicates statistical significance at 5%.

**Table 6.** The Wilks'  $\Lambda$  statistics results.

	Wilks' $\Lambda$	$F$	<i>Num DF</i>	<i>Den DF</i>
<b>1999</b>	0.72	1.43	12	119.35
<b>2000</b>	0.34	5.01*	12	119.35
<b>2001</b>	0.34	5.66*	12	119.35
<b>2002</b>	0.49	3.08*	12	119.35
<b>2003</b>	0.55	2.54*	12	119.35
<b>2004</b>	0.51	2.92*	12	119.35
<b>All</b>	<b>0.88</b>	<b>3.19*</b>	<b>12</b>	<b>807.25</b>

\* Indicates statistical significance at 5%.